ISSN 1064-2307, Journal of Computer and Systems Sciences International, 2014, Vol. 53, No. 6, pp. 939–947. © Pleiades Publishing, Ltd., 2014.
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CONTROL IN ORGANIZATIONAL AND SOCIAL–ECONOMIC SYSTEMS

Dynamic Models of Struggle against Corruption in Hierarchical Management Systems of Exploitation of Biological Resources

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Received September 12, 2013; in final form, June 30, 2014

Abstract—Dynamic models of corruption in two- and three-level management systems as applied to the optimal exploitation of biological resources are considered. "Genetic" series of corruption models are constructed. Toy examples are used to illustrate some methods of struggle against corruption.

DOI: 10.1134/S1064230714060112

INTRODUCTION

Corruption is an important social and economic phenomenon that negatively affects the development of society. In modern Russia, corruption is one of the main threats to successful social reforms. The number of publications devoted to dynamic models of corruption in hierarchical management systems is not large. Typically, these are multistep game-theoretic models in which the dynamics of the system is not explicitly described. For example, in one of the first works in this field, a recurrent statement of the problem is considered which takes into account the fact that, if an inspector strikes a bargain with the inspected person, he can be caught himself; as a result, he can also be involved in the corruption chain, but this time as a briber. The corruption chain can be finite or infinite. It was shown in [1] that, under some conditions, the increase of the probability of punishment has greater effect in the struggle against corruption than the increase of the fine. In [2], a two-step *principal–inspector–agent* model is considered, in which the principal uses the illegal bargains between the inspector and agent to gain more in the long term than in the short term. In [3], a structural analysis of corruption in the field of licensing Chinese enterprises based on the repeated bargaining model is given. It is shown that if the negotiation power is determined appropriately, some institutional features of the Chinese licensing system make corruption a structurally stable result. An analysis of comparative statics also showed that some anticorruption measures can have contrintuitive consequences. The competition between the licensing agencies is proposed as a proper measure of struggle against corruption. In [4], the "chain" of bureaucratic bodies that a businessman must pass to approve a project is studied. In the one-step statement, no projects are approved; for this reason, a multistep model is considered. Equilibriums in "trigger" strategies that minimize social losses due to corruption are described in detail. As a result, the effect of such anticorruption measures as the one stop principle and rotation of officials can be estimated. In [5], the influence of the availability of natural resources on corruption and its dependence on the quality of democratic institutions are studied. Using game-theoretic model, it is shown that the resource rent increases corruption only if the democratic institutions are weak. The paper [6] studies the guilt aversion in officials based on a repeated psychological game model. The dependence of officials' behavior and the lobbying structures on public expectations, which can be constant or can depend on time, is investigated. A separate line of research is formed by the papers [7, 8], where economic growth models with regard to corruption are studied.

The present paper is a sequel to [9] and to the studies [10–12], where static models of the struggle against corruption were investigated. Here, we consider dynamic models of this phenomenon. Our approach to the description of corruption and methods of struggling against it are based on the following points.

1. The basic scheme for the mathematical modeling of corruption is the hierarchical system *principal–supervisor–agent–object* in various modifications and its investigation using the methods of optimal control and dynamic game theory. The middle level of management (supervisor) is prone to corruption, while the upper level (the principal) is assumed to be incorruptible, and it plays the role of corruption fighter. In some cases, the simplified *bribe taker–briber* (supervisor–agent–object) model in considered in which the upper incorruptible level (principal) is taken into account implicitly (parametrically).

2. Both the leader player (principal or supervisor) and the follower player (supervisor or agent, respectively) use compulsion (mainly administrative and legislative actions) and impulsion methods (mainly economic actions) to achieve their goals. In the case of mathematical formalization, compulsion is interpreted as the influence of the leading player on the set of admissible strategies of the follower player (typically, without feedback), and impulsion is interpreted as the influence of the leading player on the payoff function of the follower player (typically, with a feedback) [13].

3. We differentiate between the administrative corruption, under which some administrative requirements are relaxed for a bribe, and economic corruption, under which a bribe makes it possible to relax economic requirements imposed by the upper management layer. In the mathematical model, administrative corruption is implemented as compulsion with the bribe size feedback, while the economic corruption is implemented as impulsion with the additional bribe size feedback.

4. Corruption exists in the forms of capture and extortion. In the case of capture, the set of services established by law is guaranteed, but additional relaxation of restrictions is provided for a bribe. In the case of extortion, a bribe is required to guarantee the basic set of services; otherwise, the requirements are made tougher.

5. For the description of corruption in hierarchical management systems with regard to the sustainable development requirement, the descriptive and normative approaches can be used. Within the descriptive approach, the functions of the administrative and economic corruption are assumed to be known, and the main problem is to identify their parameters based on statistical data. Within the normative approach, the form of the bribery function is to be determined by solving an optimization or game problem.

6. Corruption in the *principal–supervisor–agent–object* system can be investigated from three viewpoints. If the bribery function is known, then, from the viewpoint of the agent, the corruption problem can be formulated as an optimal control problem. From the viewpoint of the supervisor, this is Germeier's hierarchical parametric game of the form Γ_{2t}; its solution in the form of the corruption function with the bribe size feedback is known in the general form (see [14]). From the viewpoint of the principal, the struggle against corruption is to find the control parameters such that the resultant optimal strategy of the supervisor satisfies the sustainable development requirements for the dynamic system (object) under control.

7. It is reasonable to consider "genetic" series of more and more complicated models that more and more accurately describe the real-life corruption phenomena in hierarchical management systems. The basic logical scheme of such a complication is the *optimal control models–dynamic hierarchical two-person games–dynamic hierarchical three-person games*. With regard to various modifications of the models of each type, these series become "genetic networks."

This is exactly the last principle that determines the structure of the present paper.

1. THE SYSTEM OF DYNAMIC CORRUPTION MODELS

Let us consider the genetic series of models using economic corruption as an example. For ease of interpretation and without significant loss of generality, we will discuss the problems of optimal exploitation of biological resources [15]. In this case, the equation of dynamics of the controlled system subject to initial conditions can be written as

$$\frac{dx}{dt} = h(x(t)) - u(t)x(t), \quad x(0) = x_0. \tag{1.1}$$

Here, $x(t)$ is the biomass of the exploited population (e.g., fish population), $u(t)$ is the fraction of catch (at the time t), h is a function describing the dynamics of the homogeneous population growth (e.g., Malthus, Verhulst–Pearl, Ricker, etc.), and x_0 is the biomass at the initial time. As an example of the function $h(x(t))$, we consider the linear Malthus function $h(x(t)) = \varepsilon x(t)$, where ε is a constant.

The total revenue from fishing is aux, where a is the cost of a unit of biomass (for simplicity, we assume that it is independent of time). We assume that the fraction r of this revenue (where r can be interpreted as the tax rate with account for the possible corruption) goes to the principal (state) and the fraction $1 - r$ goes to the agent (fishery). Other interpretations of the economic impact of the variable r are possible, for example, fine. Finally, the agent gives the fraction b of its additional revenue to the supervisor as a bribe for decreasing the tax rate from the legislatively established quantity r_0 to r. Thus, the objective function of the agent can be written as

$$J_A(r(\cdot), u(\cdot), b(\cdot), x(\cdot)) = \int_0^T (a(1 - r(t))u(t) - ab(t)(r_0 - r(t))u(t) - cu^2(t))x(t)dt \to \max. \tag{1.2}$$

Here, $cu^2(t)x(t)$ are the expenses of the agent for fishing the amount of biomass $u(t)$, T is the time period under consideration, $a(r_0 - r(t))u(t)x(t)$ is are the tax arrears (the quantity the agent saves by paying only a part of the tax to which the supervisor shuts his eyes for the bribe), and $ab(t)(r_0 - r(t))u(t)x(t)$ is the cut of the agent to the supervisor.

In the case of economic corruption, the tax rate is a function of the fraction of bribe that determines the cut size. From the agent's point of view, this function is assumed to be given. For example, one can use the nonlinear capture function

$$r(b(t)) = r_0(1 - b^k(t)), \quad k = \text{const}. \tag{1.3}$$

Here, the actual tax rate is on the left hand side, the fist term on the right-hand side r_0 is its legislatively established tax rate, and the second term is its relaxation for the bribe. In particular, $r(0) = r_0$, $r(1) = 0$. Below, we consider the case $0 < k \le 1$.

Substitute (1.3) into (1.2) to obtain

$$J_A(r(\cdot), u(\cdot), b(\cdot), x(\cdot)) = \int_0^T (a(1 - r_0 + r_0 b^k(t) - r_0 b^{k+1}(t))u(t) - cu^2(t))x(t)dt. \tag{1.4}$$

Impose the constraints

$$0 \le b(t) \le 1, \quad 0 \le u(t) \le 1 \tag{1.5}$$

on the control to obtain optimal control problem (1.1), (1.4), (1.5). Instead of (1.3), other economic corruption functions may be used. The identification of the function $r(b)$ based on the actual data about corruption is of great interest. However, we leave this difficult problem for further research.

From the supervisor's point of view, the function $\tilde{r}(t) = r(b(t))$ is sought as a solution of the game Γ_{2t} between the supervisor and the agent [14]. The supervisor's objective function can be written as

$$J_S(p(\cdot), r(\cdot), u(\cdot), b(\cdot), x(\cdot)) = \int_0^T ([p(t)r(t) + b(t)(r_0 - r(t))]au(t)x(t) - K\mu(r_0 - r(t)))dt \to \max, \tag{1.6}$$

where p is the fraction of the total revenue of the principal from fishing that it gives to the supervisor (the fishery official) as his salary (or bonus), while $1 - p$ remains for the principal.

Objective function (1.6) is considered subject to the constraints on the control

$$0 \le r(t) \le r_0. \tag{1.7}$$

The second term in the integrand in (1.6) is the fine imposed on the supervisor if the tax arrears are detected, where K is the fine size and μ is the probability of detection. The values of K and μ characterize the capabilities of the principal in the struggle against corruption (i.e., the capabilities in ensuring the condition $r(t) = r_0$) by impulsion (economic influence on the supervisor's payoff function). On the whole, relations (1.1), (1.2), (1.5)–(1.7) determine a differential hierarchical two-person game of the form Γ_{2t}.

Finally, the principal's objective function can be written as

$$J_P(p(\cdot), r(\cdot), u(\cdot), b(\cdot), x(\cdot)) = \int_0^T [au(t)x(t)(1 - p(t))r(t) - M(x(t) - \bar{x})^2]dt \to \max. \tag{1.8}$$

Here, $\bar{x}$ is the biomass value that is optimal for the stable state of the population and M is the constant of the fine imposed on the principal when the current value of the biomass differs from the optimal one.

We assume that the principal uses impulsion, which corresponds to the control variable

$$0 \le p(t) \le 1. \tag{1.9}$$

The first term in the integrand in (1.8) is the principal's revenue, and the second term is the fine imposed on the principal when the stable development requirement is violated.

On the whole, relations (1.1), (1.2), (1.5)–(1.9) determine a differential hierarchical three-person game. Its information pattern depends on the control methods used by the principal. If the principal uses impulsion, then the information pattern is Γ_{1t} or Γ_{2t}, depending on whether the function $p(t)$ depends only on time or also on the supervisor's control. The game between the supervisor and the agent always has the form Γ_{2t} (see [14]) because the corruption description requires the presence of the bribe size feedback.

Thus, using the exploitation of biological resources as an example, we have determined the sequence of dynamic economic corruption models in the form *optimal control problem–dynamic hierarchical two-person game–dynamic hierarchical three-person game*. Below, we describe algorithms for the investigation of these models. We assume that only impulsion is used in the system.

2. SINGLE-LEVEL MODEL

Consider the economic corruption model described by relations (1.1), (1.4), (1.5). Function (1.2) defines the income of a fishery, which can be a briber. The fishery pays taxes from this revenue and also possibly bribes an official. The tax size depends on the bribe size—the greater the bribe, the lower is the tax. If the bribe is zero (there is no bribe), then the tax is paid based on the tax rate established by the law. It is clear that the tax reduction for the bribe must compensate for the expenses needed to give the bribe; otherwise, the corrupted deal will be unprofitable for the fishery. Hence, we have an optimal control problem, which can be solved using Pontryagin's maximum principle. First, our consideration will be within the domain of feasible controls, and then we will take into account constraints (1.5).

The Hamiltonian for problem (1.1), (1.4), (1.5) is

$$H(u(t), b(t), y(t), x(t)) = a(1 - r_0 + r_0 b^k(t) - r_0 b^{k+1}(t))u(t)x(t)$$

$$- cu^2(t)x(t) + y(t)x(t)(\varepsilon - u(t)),$$

where x is the state variable, u and b are the control variables, and y is the adjoint variable (a function of time). We have

$$\frac{\partial H}{\partial b} = r_0 a u(t)x(t)b^{k-1}(t)(k - (k+1)b(t)) = 0,$$

$$\frac{\partial H}{\partial u} = (1 - r_0 + r_0 b^k(t) - r_0 b^{k+1}(t))ax(t) - 2cx(t)u(t) - y(t)x(t) = 0;$$

that is, the potential maximum within domain (1.5) is

$$b^0(t) = \frac{k}{k+1}, \quad u^0(t) = \frac{A - y(t)}{2c}, \tag{2.1}$$

where

$$A = a\left(1 - r_0 + \frac{r_0}{k+1}\left(\frac{k}{k+1}\right)^k\right),$$

$$-\frac{\partial H}{\partial x} = \frac{dy}{dt} = cu^2(t) - \left(1 - r_0 + \frac{r_0}{k+1}\left(\frac{k}{k+1}\right)^k\right)au(t) + (u(t) - \varepsilon)y(t), \quad y(T) = 0. \tag{2.2}$$

Plug (2.1) into (2.2) and solve the resulting differential equation with separating variables to obtain

$$T - t = \int_0^T \frac{dy}{B + Ey + 0.25y^2/c} = \begin{cases} \frac{2}{D}\arctan\frac{y/(2c) + E}{D}\Big|_0^y & \text{if} \quad E^2 < B/c \\ \frac{1}{D}\ln\frac{y/(2c) + E - D}{y/(2c) + E + D}\Big|_0^y & \text{if} \quad E^2 > B/c, \\ -\frac{1}{d}\frac{1}{y + 2Ec}\Big|_0^y & \text{if} \quad E^2 = B/c, \end{cases}$$

where $B = 0.5A^2(a - 0.5)/c$; $E = \varepsilon - 0.5Aa/c$; $D = \sqrt{|B/c - E^2|}$. Hence,

$$y(t) = \begin{cases} 0.5c\left(D\mathrm{tg}(0.5D(T - t) + \arctan(E/D)) - E\right) & \text{if} \quad E^2 < B/c, \\ 0.5c\left(2D\frac{E + D}{E - D}e^{-D(T-t)} - E - D\right) & \text{if} \quad E^2 > B/c, \\ 4cE^2(T - t)/(2 - TE + Et) & \text{if} \quad E^2 = B/c. \end{cases} \tag{2.3}$$

Plug (2.3) into (2.1). By calculating the Hessian at point (2.1), we obtain

$$\frac{\partial^2 H}{\partial b^2}\Big|_{(u^0(t), b^0(t))} = -kr_0 a u^0(t)x(t)(b^0(t))^{k-2},$$

$$\frac{\partial^2 H}{\partial u^2}\Big|_{(u^0(t), b^0(t))} = -2cx(t); \quad \frac{\partial^2 H}{\partial u \partial b}\Big|_{(u^0(t), b^0(t))} = 0.$$

Taking into account the negative definiteness of the Hessian in (2.1) and constraints (1.5), we conclude that the optimal strategies of the subjects of control are determined for any t (with regard to (1.5), (2.1), and (2.2) by the rule

$$(u^*(t), b^*(t)) = \begin{cases} \left(u^0(t), \frac{k}{k + 1}\right) & \text{if} \quad 0 \le u^0(t) \le 1, \\ \left(1, \frac{k}{k + 1}\right) & \text{if} \quad u^0(t) > 1, \\ (0, 0) & \text{if} \quad u^0(t) < 0. \end{cases} \tag{2.4}$$

Plugging (2.3) and (2.4) into (1.4), we obtain the optimal agent's gain (examples can be found in Section 5).

3. TWO-LEVEL MODEL

Consider the economic corruption model described by relations (1.1), (1.2), (1.5)–(1.7). In the case of information extension of Germeier's game $\Gamma_{2,t}$, the initial sets of admissible strategies of the supervisor and agent

$$R = \{0 \le r(t) \le r_0\}; \quad U = \{0 \le u(t) \le 1\}; \quad B = \{0 \le b(t) \le 1\}, \quad \text{where} \quad 0 \le t \le T,$$

become more complex—now they are sets of mappings $\tilde{R} = \{\tilde{r} = r(u, b) : U \times B \to R\}$, $U^{(2)} = \{u^{(2)} = u(r(u, b)) : \tilde{R} \to U\}$.

Since the players' payoff functions are defined on the initial sets of admissible strategies, we must define the projection (see [16]) $\pi : \tilde{R} \times U^{(2)} \times B \times X \to R \times U \times B \times X$, which accounts for the information pattern of the game; the projection is obtained by the substitution of the corresponding arguments into the functions $\tilde{r}$ and $u^{(2)}$.

We propose to use the equilibrium construction algorithm described in [14]. It is as follows.

1. Calculate the maximum guaranteed gain of the agent when the supervisor uses the punishment strategy:

$$L_A = \sup_{(u^{(2)},b)\in U^{(2)}\times B} \inf_{\tilde r\in\tilde R} J_A(\pi(\tilde r, u^{(2)}, b, x)).$$

2. Find the set $D_A = \{(\tilde r, u^{(2)}, b) : J_A(\pi(\tilde r, u^{(2)}, b, x)) \geq L_A\}$.

3. Calculate the maximum guaranteed gain of the supervisor with the agent's interests taken into account:

$$L_S = \sup_{(\tilde r, u^{(2)}, b)\in D_A} J_S(\pi(\tilde r, u^{(2)}, b, x));$$

hence, we also obtain its optimal strategies

$$(r^*, u^*, b^*) \in \arg \sup_{(\tilde r, u^{(2)}, b)\in D_A} J_S(\pi(\tilde r, u^{(2)}, b, x)).$$

Certainly, this algorithm can be implemented only if $D_A \neq \varnothing$.

The supervisor's punishment strategy (Step 1 of the algorithm) is to select the maximum possible tax rate from (1.7) ($r^P = r_0$). In this case, the agent does not propose a bribe ($b^P = 0$); and the optimal catch fraction (the function $u = u(t)$) is determined by solving an optimal control problem, which is solved using Pontryagin's maximum principle. The Hamiltonian in this problem is

$$H(u(t), y(t), x(t)) = \left(a(1-r_0)u(t)x(t) - cu^2(t)x(t) + y(t)x(t)(\varepsilon - u(t))\right),$$

where the same notation as in Section 2 is used. Hence,

$$\frac{\partial H}{\partial u} = (1-r_0)ax(t) - 2cx(t)u(t) - y(t)x(t) = 0.$$

Reasoning as in Section 2, we obtain a potential maximum in domain (1.5)—the point $u^0(t) = 0.5(A - y(t))/c$,, where $A = a\left(1 - r_0\right)$. We also have

$$-\frac{\partial H}{\partial x} = \frac{dy}{dt} = cu^2(t) - (1-r_0)au(t) + (u(t)-\varepsilon)y(t); \quad y(T) = 0.$$

The solution to this differential equation has form (2.3), where $A = a\left(1 - r_0\right)$.

The second-order derivative of the Hamiltonian

$$\left.\frac{\partial^2 H}{\partial u^2}\right|_{(u^0(t), b^0(t))} = -2cx(t)$$

is less than zero in this case. Therefore, the optimal catch fraction when the supervisor uses the punishment strategy is determined for every t by the rule

$$u^P(t) = \begin{cases} u^0(t) & \text{if} \quad 0 \leq u^0(t) \leq 1, \\ 1 & \text{if} \quad u^0(t) > 1, \\ 0 & \text{if} \quad u^0(t) < 0; \end{cases}$$

furthermore,

$$L_A = \sup_{0\leq u^P\leq 1} \int_0^T (a(1-r_0)u^P(t) - c(u^P)^2(t))x^P(t)dt,$$

where

$$\frac{dx^P}{dt} = (\varepsilon - u^P(t))x(t), \quad x^P(0) = x_0.$$

The problem formulated at Steps 2 and 3 of the algorithm is solved numerically (see Section 5).

4. THREE-LEVEL MODEL

Consider the economic corruption model described by relations (1.1), (1.2), (1.5)–(1.9). The main idea of economic corruption is to change the impulsion strategy for a bribe in favor of the agent. The principal must ensure the requirements of sustainable development by motivating the supervisor. Let us define the sets related to the information structure of the problem in this case:

$$\tilde{P} = \{\tilde{p} = p(r) : R \to P\}, \quad R^{(3)} = \{r^{(3)} = r(\tilde{p}, u, b) : \tilde{P} \times U \times B \to R\},$$

$$U^{(3)} = \{u^{(3)} = u(r(\tilde{p}, u, b)) : R^{(3)} \to U\}$$

and the projection [16] $\pi_1 : P \times R^{(3)} \times U^{(3)} \times B \times X \to P \times R \times U \times B \times X$.

We propose to use the equilibrium construction algorithm based on the procedure described in [16, p. 39].

1. Calculate the maximum guaranteed gain of the agent when the principal and the supervisor use punishment strategies:

$$L_A = \sup_{(u^{(3)}, b) \in U^{(3)} \times B} \inf_{\tilde{p} \in \tilde{P}} \inf_{r^{(3)} \in R^{(3)}} J_A(\pi_1(\tilde{p}, r^{(3)}, u^{(3)}, b, x)).$$

2. Find the set

$$D_A = \{(r^{(3)}, u^{(3)}, b) : J_A(\pi_1(\tilde{p}^P, r^{(3)}, u^{(3)}, b, x)) \geq L_A\},$$

where $\tilde{p}^P$ is the strategy used by the principal to punish the supervisor, i.e.,

$$J_S(\pi_1(\tilde{p}^P, r^{(3)}, u^{(3)}, b, x)) = \min_{\tilde{p} \in \tilde{P}} J_S(\pi_1(\tilde{p}, r^{(3)}, u^{(3)}, b, x)).$$

3. Calculate the maximum guaranteed gain of the supervisor with the agent's interests taken into account, whle the principal uses the punishment strategy: $L_S = \sup_{(r^{(3)}, u^{(3)}, b) \in D_A} J_S(\pi_1(\tilde{p}^P, r^{(3)}, u^{(3)}, b, x))$.

4. Define the set

$$D_S^* = \{(\tilde{p}, r^{(3)}, u^{(3)}, b) : J_S(\pi(\tilde{p}, r^{(3)}, u^{(3)}, b, x)) \geq L_S, J_A(\pi_1(\tilde{p}, r^{(3)}, u^{(3)}, b, x)) \geq L_A\}$$

to take into account the interests of the supervisor and the principal when the sustainable development requirements under the conditions of economic corruption are met.

5. Then, the guaranteed gain of the principal is

$$\gamma_P = \sup_{(\tilde{p}, r^{(3)}, u^{(3)}, b) \in D_S^*} J_P(\pi(\tilde{p}, r^{(3)}, u^{(3)}, b, x)),$$

whence we obtain the strategies to be used in the system equilibrium state

$$(p^*, r^*, u^*, b^*) = \arg \sup_{(\tilde{p}, r^{(3)}, u^{(3)}, b) \in D_S^*} J_P(\pi(\tilde{p}, r^{(3)}, u^{(3)}, b, x)).$$

This algorithm can be implemented if $D_A \neq \varnothing, D_S^* \neq \varnothing$.

The agent's objective function is independent of the principal controls; therefore, L_A (Step 1 of the algorithm) is determined using the formulas from the preceding section. The problems at Steps 3–5 of the algorithm are solved numerically. Numerical examples are discussed in the next section.

5. NUMERICAL RESULTS

The optimal control problems in the single-level models are solved analytically (formulas (2.3), (2.4)). The equilibriums in the two- and three-level models are generally constructed numerically using the direct ordered search technique in the domain of admissible controls by analogy with [11] after the models are sampled with respect to time [17].

Example 1 (single-level model). In the case $\varepsilon = 1/720$ d^{-1}, $a = 1$ (a.u./kg), $k = 0.5$, $r_0 = 0.4$, $c = 2$ (a.u./kg), $x_0 = 1000$ kg, and $T = 360$ d (where a.u. stands for arbitrary units and d stands for day), we obtain the agent's gain $J_A = 12715$ (here and in what follows, the gain is given in arbitrary units). If the established tax rate decreases or the population growth coefficient increases, the agent's gain increases.

For example, when $r_0 = 0.1$, we have $J_A = 36087$; when $\varepsilon = 1/3600$ d^{-1}, we have $J_A = 8502$. As the exponent k determining the bribe function or the expense for fishing a unit of biomass increases, the agent's gain decreases; for $k = 1$, we have $J_A = 2402$; for $c = 1$ a.u./kg, we have $J_A = 33960$.

Example 2 (two-level model). In the case $\varepsilon = 1/720$ d^{-1}, $a = 1$ (a.u./kg), $k = 0.5$, $r_0 = 0.4$, $c = 2$ (a.u./kg), $K = 100$ a.u., and $\mu = 0.1$, it becomes unprofitable for the agent to continue its activity—its controls vanish. The gain of all the subjects also vanishes. The further examples describe the case when the fishing expenses decrease ($c = 0.5$ a.u./kg). Then, fishing is profitable under the conditions of corruption—the gain of the subjects in a.u. is, respectively,

$$L_A = 236; \quad J_A = 265; \quad J_S = 563.$$

As the established tax rate ($r_0 = 0.1$) decreases, the agent's gain increases, the supervisor's gain decreases, and there is no corruption in the system:

$$L_A = 944; \quad J_A = 951; \quad J_S = 140.$$

As the population growth coefficient increases ($\varepsilon = 0.9/360$ d^{-1}) or the probability of bribery detection decreases, the gain of both subjects under the corruption conditions increases:

$$L_A = 285; \quad J_A = 326; \quad J_S = 728 \quad \text{and} \quad L_A = 236; \quad J_A = 2765; \quad J_S = 875$$

respectively.

Example 3 (three-level model). In the case $\varepsilon = 1/720$ d^{-1}, $a = 1$ (a.u./kg), $k = 0.5$, $r_0 = 0.4$, $x_0 = 1000$ kg, $T = 360$, $K = 100$ a.u., $\mu = 0.1$, $M = 0.004$ a.u./kg, and $c = 2$ a.u./kg, both the agent and the principal gain in the case of corruption, the supervisor suffers losses, and

$$L_A = 236; \quad L_S = -143; \quad J_A = 245; \quad J_S = -102; \quad J_P = 290.$$

If the fine imposed on the supervisor for deviating from the established tax rate decreases ($K = 50$ a.u.), the gains of the agent and the supervisor under corruption increase, while the principal's gain decreases:

$$L_A = 236; \quad L_S = 37; \quad J_A = 281; \quad J_S = 41; \quad J_P = 221.$$

If the cost of fishing decreases ($c = 0.1$ a.u./kg) or the tax rate decreases ($r_0 = 0.1$), the agent's gain increases, while the supervisor's and principal's gain decreases:

$$L_A = 1180; \quad L_S = -38; \quad J_A = 1198; \quad J_S = -20; \quad J_P = -711.$$

On the contrary, if the amount of the biomass that is optimal for the stable state of the population decreases ($\bar{x} = 1050$ kg), then the agent's gain decreases, while the supervisor's and principal's gain increases:

$$L_A = 236; \quad L_S = 37; \quad J_A = 241; \quad J_S = 43; \quad J_P = 329.$$

Thus, the investigation in the framework of the proposed models showed that the successful struggle against corruption is possible either by decreasing the tax rate or by strengthening the control over the supervisor.

CONCLUSIONS

General principles of constructing dynamic models of struggling against corruption are proposed. A genetic series of dynamic models of economic corruption in the form *optimal control problem–dynamic hierarchical two-person game–dynamic hierarchical three-person game* as applied to the optimal exploitation of biological resources is constructed. The consideration is based on the concept of hierarchically controlled dynamic systems and the concept of impulsion equilibrium with regard to the sustainable development requirement including the case of corruption [18]. Algorithms for obtaining these equilibriums based on the results obtained in [14, 17] are proposed.

In the future, we plan to investigate the models of economic corruption for various corruption functions and functions of the dynamics of the managed system (using available statistical data). Also, we are going to investigate dynamic models of administrative corruption.

ACKNOWLEDGMENTS

This work was supported by the Russian Foundation for Basic Research (project no. 12-01-00017) and by the Southern Federal University.

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Translated by A. Klimontovich